**Computer Science & Engineering Department**
**I. I. T. Kharagpur**

**Foundations of Computing: CS30053**
*3rd Year : Autumn Semester*
*Class Test II* (**Total Marks: 20**)

From 1730hr to 1830hr *Date : 28th October, 2003*

**Answer All Questions**
**Do not write *illogical* statements.**

1. Justify that a finte language $L$ (there are finite number of strings in $L$) is always a *context-free* language. [**2**]

2. Consider the context-free grammar $G = (\{A, B\}, \{0, 1\}, R, A)$, where

$$R = \{A \to BB,\ B \to BB,\ B \to 0,\ B \to 1B,\ B \to B1\}.$$

[$\mathbf{2+2+1}$]

   (a) List all strings of $L(G)$ produced in **three (3)** or fewer steps of derivations.

   (b) Give two parse trees of derivations corresponding to the string '**0010**'.

   (c) What is your conclusion about the grammar?

3. Give context-free grammars for the following languages. Write short explanation about the production rules. [$\mathbf{5+5}$]

   (a) $L_1 = \{a^m b^n c^p d^q \ : \ m + n = p + q\}$.

   (b) $L_2 = \{uaxb \ : \ u, x \in \{0, 1\}^*,\ |u| = |x|\}$.

4. Let $\Sigma$ and $\Gamma$ be two alphabets and

$$h : \Sigma \longrightarrow \Gamma^*$$

be a map so that **each element** of $\Sigma$ is mapped to a **string** over $\Gamma$. Let $L$ be a language over $\Sigma$.

$$h(L) = \{y \in \Gamma^* \ : \ \exists x \in L, \text{so that } x = \sigma_1 \cdots \sigma_n \text{ and } y = h(\sigma_1) \cdots h(\sigma_n)\}.$$

Each $\sigma_i \in \Sigma$, $1 \leq i \leq n$. The value of $h(\varepsilon) = \varepsilon$.

Justify that $h(L)$ is a context-free language if $L$ is context-free. [**3**]