INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Date .... FN / AN *Time:* 2/3 Hrs. *Full Marks* .... *No. of Students* ....
Autumn / Spring Semester, 20 .... Deptt. .... Sub No. ....
...... Yr. B. Tech.(Hons.) / B. Arch. / M. Sc. Sub. Name ....

Instructions : **Answer Q1. & Any Three (3) from the Remaining Questions (2-5). Do not write illogical statements.**

1. $[\mathbf{10 \times 3 = 30}]$

Answer with a short justification whether the following statements are *true* or *false*. No credit will be given for writing only *true* or *false*.

(a) Every language over the alphabet $\{0, 1\}$ is **Turing Recognisable (acceptable)**.

(b) Every **Turing Decidable** language is a **context-free** language.

(c) Some context-free languages may have both unambiguous as well as ambiguous grammars.

(d) Any *non-null* string x of a context-free language can always be **derived** in $2n - 1$ steps (using a suitable grammar), where $|x| = n$.

(e) If L_1 and L_2 are context-free languages, then so is

$$L = L_1 L_2 = \{w \;:\; \exists x \in L_1,\ \exists y \in L_2,\ w = xy\}.$$

(f) The following language is **not Turing Decidable**

$$L_{\phi}^{CFG} = \{< G >:\ G \text{ is a CFG and } L(G) = \phi\}.$$

(g) The following language is **Turing Recognisable (acceptable)**

$$L_{R}^{TM} = \{< M, x >:\ M \text{ is a Turing Machine and } M \textbf{ does not accept } x\}.$$

(h) A **linear bounded automaton** can be used as a **decider** of the language $L = \{a^n b^n c^n \;:\; n > 1\}$.

(i) If a language L is **polynomial time mapping reducible** to the language L' and L is **NP-complete**, then so is L'.

(j) If $\mathbf{P} = \mathbf{NP}$ then every language $L \in P$ except $L = \emptyset$ and $L = \Sigma^*$ is **NP-complete**.

2. $[\mathbf{7 + 8}]$

(a) Use Cocke-Kasami-Younger (CKY) algorithm to show that **aabbab** can be generated by the grammar $G = (\{S, A, B, C, D\}, \{a, b\}, R, S)$, where

$$\begin{aligned} S &\rightarrow AB \mid BA \mid AC \mid BD \mid SS \\ C &\rightarrow SB \\ D &\rightarrow SA \\ A &\rightarrow a \text{ and } B \rightarrow b \end{aligned}$$

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(b) Give a context-free grammar for the following language and convert the grammar to Chomsky Normal Form: $\mathbf{L} = \{\mathbf{a^k b^m c^n} : \mathbf{k, m \geq 1,\ k \leq n \leq 2k}\}$.

3. **[10 + 5]**

(a) Design a **single tape deterministic Turing machine** $M = (Q, \Sigma, \Gamma, \delta, q_0, q_A, q_R)$ that accepts the following language in $\mathbf{O(n \log n)}$ time : $L = \{a^n b^n \ : \ n \geq 0\}$. Clearly specify each component of M and give the state transition function δ as a state transition diagram (give short explanation for each transition). Assume that there is a special **marker** $\$ \notin \mathbf{\Sigma}$ at the leftmost square of the tape.

(b) Give the outline of proof of the following statement:
$L = \{< G > : \ G$ is a CFG over $\{0,1\}$ and $\{1\}^* \cap L(G) \neq \emptyset\}$ is a **decidable language**.

4. **[3 + 7 + 5]**

(a) Prove that the following language is **Turing Recognisable (acceptable)**:
$L^{TM}_{HALT} = \{< M, x > : \ M$ is a Turing machine and M **halts** on $x\}$.

(b) Use **diagonalisation** and **reduction to a contradiction** (not the reduction of another language to L^{TM}_{HALT}) to prove that the language L of **(4a)** is **undecidable**.

(c) Let L_1 and L_2 be two disjoint languages. Say that language L **separates** L_1 and L_2 if $L_1 \subseteq L$ and $L_2 \subseteq \overline{L}$. Show that two disjoint **co-Turing-Recognisable** languages are separated by some **decidable language**.

5. **[4 + 7 + 4]**

(a) Give justification for the following statement:
All deterministic Turing machine models are polynomially equivalent.

(b) Prove that the following language

$$TRIANGLE = \{< G > : \ G \text{ is a undirected graph and } G \text{ has a } \textbf{3-clique}\},$$

is in the complexity class **P**. A 3-clique is a complete subgraph of three vertices.

(c) **3SAT** is known to be an NP-complete language. Show by polynomial time mapping reduction that **VERTEX-COVER** is also an NP-complete language.

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