INDIAN INSTITUTE OF TECHNOLOGY, KHARAGPUR

Date ....FN / AN *Time:* 2/3 Hrs. *Full Marks* .... *No. of Students* ....
Autumn / Spring Semester, 20 .... Deptt. .... Sub No. ....
......Yr. B. Tech.(Hons.) / B. Arch. / M. Sc. Sub. Name ....

Instructions : **Answer All (4) Questions. Do not write illogical statements.**

1. $[\mathbf{2+2+3+3+2+3+3+2+2+3=25}]$

Answer with a short justification whether the following statements are *true* or *false*. No credit will be given for writing only *true* or *false*.

(a) A set may be equinumerous to its proper subset.

(b) No *left-inverse* of a map $f : A \longrightarrow B$ is present if it is not *one-to-one*.

(c) Given the map $f : A \longrightarrow B$, we define a map $F : \mathcal{P}B \longrightarrow \mathcal{P}A$, so that $F(D) = \{c \in A \ : \ f(c) \in D\}$, $D \subseteq B$. The map F is monoton i.e. $C \subseteq D \Rightarrow F(C) \subseteq F(D)$.

(d) The collection of all finite subsets of natural numbers, $\mathbf{2}^{\mathbb{N}}_{fin} = \{A : \ A \subset \mathbb{N} \text{ and } A \text{ is finite}\}$, is not denumerable.

(e) A denumerable set of variables can be defined inductively using a *finite alphabet*.

(f) Fifteen (15) different equivalence relations can be defined on a set of four elements $A = \{a, b, c, d\}$. [Note that an equivalence relation divides the set into nonempty parts.]

(g) The λ-term $(\lambda xyz.xz(yz))(\lambda ab.a)(\lambda p.pp)(\lambda xy.y)$ can be reduced to '**false**'.

(h) Reflexive-transitive closure of a binary relation R on a set A is always an equivalence relation.

(i) Every β-reduction of any λ-term will give a β-normal form.

(j) The decision problem, "the vertex **d** is reachable from the vertex **s** in a undirected graph G", can be translated to a decision problem of a language.

2. $[\mathbf{5}]$

Let the collection of C* programs that calculate functions on the set of natural numbers, $\mathbb{N} \to \mathbb{N}$, be $\mathcal{C}_{\mathbb{N}}$. Prove by direct use of Cantor's diagonal argument that $\mathcal{C}_{\mathbb{N}}$ cannot be *equinumerous* to the collection of all functions on natural numbers ($\mathbb{N}^{\mathbb{N}}$).

3. $[\mathbf{2+2+1}]$

Give an example of a *fixed point combinator* other than the Curry and the Turing combinators. Show that it is a *fixed point combinator*. Justify that there are infinite many of them.

4. $[\mathbf{10}]$

Give an inductive definition of '**remainder m n**' ($m\%n$). Justify that *remainder* is λ-definable. Start from the *Barendregt numeral* and define everything that is required to define *remainder*.

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