Indian Association for the Cultivation of Science

(Deemed to be University under *de novo* Category)

Master's/Integrated Master's-PhD Program/ Integrated Bachelor's-Master's Program/PhD Course

Mid-Semester (Sem-III) Examination-Autumn 2025

Subject: **Theory of Computation II** *Subject Code:* **COM 5108**

Full Marks: **25** *Time Allotted:* **2 hours**

Q 1. Answer five (5) questions with brief justifications. Only '*Yes*' or '*No*' will not be sufficient. **[5 × 2 = 10]**

(a) Is it true that $\binom{n}{30}$ is of $O(n^k)$ for some k?

(b) Let $\mathcal{P}\mathbb{N}_{cofin} = \{A \subseteq \mathbb{N}_0 : \mathbb{N}_0 \setminus A \text{ is finite}\}$, the collection of *cofinite* (complement is finite) subsets of $\mathbb{N}_0 = \{0, 1, 2, \cdots\}$.
Is $\mathcal{P}\mathbb{N}_{cofin}$ countably infinite?

(c) Let L_1 and L_2 be two *Context-free languages* (CFLs) over Σ.
$L = L_1L_2 = \{x \in \Sigma^* : x = uv, \ u \in L_1 \wedge v \in L_2\}$. Is L a CFL?

(d) Let L_1 and L_2 be two decidable languages over Σ.
$L = L_1L_2 = \{x \in \Sigma^* : x = uv, \ u \in L_1 \wedge v \in L_2\}$. Is L decidable?

(e) $L_{1000} = \{< M, x >$: M is a Turing machine that does not halt on input x within 1000 steps $\}$. Is L_{1000} decidable?

(f) A Boolean clause has 10 literals, $C = (l_1 \vee l_2 \vee \cdots \vee l_{10})$. How many clauses in the equivalent 3CNF formula ϕ will be there so that C is satisfiable if and only if ϕ is satisfiable? How many new Boolean variables are introduced?

(g) Let $CLIQUE_{30} = \{< G >$: G is an undirected graph and it has a clique of size 30 $\}$. Is $CLIQUE_{30}$ in **P**?

Answer any three (3) of the following questions.

[3 × 5 = 15]

Note: *decidable ≡ recursive, semi-decidable ≡ recursively-enumerable.*

Q 2. **[2 + 3]**

(a) Prove that for any set A there is no *onto* map from A to its *power set* $\mathcal{P}A$.

(b) Let $L_H = \{< M, x >$: M is a Turing machine that halts on its input $x\}$.
Prove that L_H is undecidable.

→ **See page 2**

Q 3. **[2 + 3]**

(a) Let $A, B \subseteq \{0,1\}^*$. Define *mapping reducibility* ($\leq_m$) from A to B. What conclusions can you draw about the relative computability (*decidability*, semi-decidability etc.) between A and B?

(b) Show that $\overline{L_H}\{< M, x >:$ the Turing machine M does not halt on $x\}$ is mapping reducible to $\overline{L_{reg}} = \{< M >: L(M)$ is not a regular language$\}$. Is $\overline{L_{reg}}$ *recursive* or *recursively-enumerable* or *not recursively-enumerable*?

Q 4. **[4 + 1]**

(a) Give two definitions of the class **NP** and show that they are equivalent.

(b) Is $COMPOSITE = \{n \in \mathbb{N} : n$ is a composite number$\}$ in **P**?

Q 5. **2 + 3**

(a) Define the class of search problems **PF** (solution can be found in polynomial time) and the class **PC** (solution can be verified in polynomial time).

(b) Prove that, if **PC** $\subseteq$ **PF** then **NP** $=$ **P**.

*** **End** ***