**School of Mathematical and Computational Sciences**

# Indian Association for the Cultivation of Science

*Master's/Integrated Master's-PhD Program/ Integrated Bachelor's-Master's Program/PhD Course*

## Theory of Computation II: COM 5108

*Tutorial I (06 August 2025)*

*Instructor:* Goutam Biswas *Autumn Semester 2023*

1. Find a *bijection* between $[a, b]$ and $[-\pi/2, \pi/2]$. Give a proof.
2. Find a *bijection* between $(-\pi/2, \pi/2)$ to $(-\infty, \infty)$.
3. Find a *bijection* between $(a, b)$ and $(-\infty, \infty)$,
4. Prove that there is no bijection between a set $A$ and its power set $\mathcal{P}A$.
5. Show that there no *bijection* from $\mathbb{N}_0 = \{0, 1, 2, 3, \cdots\}$ to the collection of all functions from $\mathbb{N}_0 \to \mathbb{N}_0$.
6. Show that there is a *bijection* between $\mathbb{N}_0 \times \mathbb{N}_0$ to $\mathbb{N}_0$, where $\mathbb{N}_0]\{0, 1, 2, 3, \cdots\}$.
7. Give an informal description of a single-tape Turing machine that decides the language $L_1 = \{0^n 1^n : \ n \in \mathbb{N}_0\}$, where $\mathbb{N}_0 = \{0, 1, 2, \cdots\}$, in time $O(n \log n)$.
   Can this be done in time $o(n \log n)$?
8. Give an informal description of a 2-tape Turing machine that decides the language $L_1 = \{0^n 1^n : \ n \in \mathbb{N}_0\}$ in time $O(n)$.