School of Mathematical and Computational Sciences

Indian Association for the Cultivation of Science

Master's/Integrated Master's-PhD Program/ Integrated Bachelor's-Master's Program/PhD Course

Theory of Computation II: COM 5108

Tutorial II (13 August 2025)

Instructor: Goutam Biswas *Autumn Semester 2025*

1. Let $\mathcal{P}\mathbb{N}_{fin} = \{A \subset \mathbb{N}_0 : \ A \text{ is finite}\}$. Prove that there is a *bijection* between $\mathbb{N}_0$ and $\mathcal{P}\mathbb{N}_{fin}$.
2. Show that there is no bijection between $\mathbb{N}_0$ and $\mathbb{R}$.
3. Is $L_{DFA} = \{< D, x >: \ D \text{ is a DFA and } D \text{ accepts } x\}$ a decidable language?
4. The language $L_H = \{< M, x >: \text{ The TM } M \text{ halts on } x\}$ is turing *recognizable*.
   Show that if $\overline{L_H} = \{< M, x >: \text{ The TM } M \text{ does not halts on } x\}$ is also Turing *recognizable* then L_H is Turing *decidable*.
5. Let $L_1, L_2 \subseteq \{0,1\}^*$ be Turing recognizable languages.
   (a) Show that $L_1 \cup L_2$ is Turing recognizable.
   (b) Show that $L_1L_2 = \{x \in \{0,1\}^* : \ \exists u, v, \ x = uv, \ u \in L_1, v \in L_2\}$ is Turing recognizable.

   What is your conclusion?
6. Show that $L_w = \{< M >: w \in L(M)\}$ is undecidable.