
**School of Mathematical and Computational Sciences**

# Indian Association for the Cultivation of Science

*Master's/Integrated Master's-PhD Program/ Integrated Bachelor's-Master's Program/PhD Course*

## Theory of Computation II: COM 5108

*Tutorial IV (03/10 September 2025)*

*Instructor:* Goutam Biswas *Autumn Semester 2025*

1. Consider the following *truth table* and give the corresponding CNF formula.

| $a$ | $b$ | $c$ | $d$ | $f$ |
|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 1 | 1 |
| 0 | 0 | 1 | 0 | 1 |
| 0 | 0 | 1 | 1 | 1 |
| 0 | 1 | 0 | 0 | 1 |
| 0 | 1 | 0 | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 |

| $a$ | $b$ | $c$ | $d$ | $f$ |
|---|---|---|---|---|
| 1 | 0 | 0 | 0 | 0 |
| 1 | 0 | 0 | 1 | 1 |
| 1 | 0 | 1 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 |
| 1 | 1 | 0 | 0 | 1 |
| 1 | 1 | 0 | 1 | 1 |
| 1 | 1 | 1 | 0 | 0 |
| 1 | 1 | 1 | 1 | 1 |

2. Convert the first two of the clauses of **Q.1** to 3CNF form. Justify your answer.

3. Justify that all $n$-variable Boolean functions cannot be computed by $poly(n)$ size Boolean circuits.

4. (a) Draw the *implication graph* corresponding to the following 2SAT formula and give a satisfying assignment if possible.

$$(a \vee \neg b) \wedge (b \vee \neg c) \wedge (c \vee \neg d) \wedge (\neg a \vee c) \wedge (\neg b \vee d) \wedge (\neg c \vee \neg d).$$

   (b) What will happen if $(a \vee \neg b)$ is replaced by $(a \vee b)$?

5. How many clauses and how many literals per clause will be there in the equivalent CNF formula for

$$\phi = \bigvee_{i=1}^{n} (x_i \wedge y_i).$$

6. Construct a relation $R \subseteq \{0,1\}^* \times \{0,1\}^*$ that does not belong to **PC**?