School of Mathematical and Computational Sciences

Indian Association for the Cultivation of Science

Master's/Integrated Master's-PhD Program/ Integrated Bachelor's-Master's Program/PhD Course

Theory of Computation II: COM 5108

Tutorial V (08 October 2025)

Instructor: Goutam Biswas *Autumn Semester 2025*

1. Reduce R_{3COL} to $3COL$ where

   $$3COL = \{< G >:\ G \text{ is a 3-colourable (vertex) undirected graph.}$$

   R_{3COL} is the corresponding colouring relation.

2. Consider the Boolean circuit $((a \wedge \neg b) \vee c) \wedge (\neg c \vee d)$. We need to convert it to 3CNF formula.

   (a) Let m be the number of gate-variables to convert them to m constant size formulae. What is the value of m?

   (b) Give three constant size formula for $(a \wedge \neg b) \vee c$.

   (c) Give the 3CNF formula corresponding to $(a \wedge \neg b) \vee c$.

3. Prove that

   $$INDSET = \{(G, k) :\ G \text{ is a graph, } \exists S \subseteq V(G) \text{ s.t. } |S| \geq k \text{ and } \forall u, v \in S,\ (u, v) \notin E(G)\}.$$

   is **NP**-*complete*.