



**School of Mathematical and Computational Sciences
Indian Association for the Cultivation of Science**

*Master's/Integrated Master's-PhD Program/ Integrated
Bachelor's-Master's Program/PhD Course*

Theory of Computation II: COM 5108

Tutorial VI (15 October 2025)

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Exercise 1. We have already proved that

$INDSET = \{ \langle G, k \rangle : \text{the undirected graph } G \text{ has an independent set of vertices of size } k \}$

is **NP-complete**. Use this result to prove that the following two sets are also **NP-complete**.

- (a) $VERTEX - COVER = \{ \langle G, k \rangle : G \text{ is a undirected graph, } S \subseteq V(G), |S| = k \text{ and } \forall e \in E(G) \exists v \in S \text{ such that } e \text{ incident on } v \}$.
- (b) $CLIQUE = \{ \langle G, k \rangle : G \text{ is a undirected graph which has a } \textit{clique} \text{ of size } k \}$.

Exercise 2. (*Integer Linear Programming (IP)*) Prove that the following problem is **NP-complete**. An instance of the problem is a system of linear inequalities with integer constants and the problem is to determine whether the system has an integer solution.

$$\sum_{i=1}^n a_i y_i \geq b_i, \quad a_i \text{ and } b_i \text{ are integer constants.}$$

We can restrict the solution space to $\{0, 1\}$ by adding the inequalities $x \geq 0$ and $-x \geq -1$.