School of Mathematical and Computational Sciences

Indian Association for the Cultivation of Science

Master's/Integrated Master's-PhD Program/ Integrated Bachelor's-Master's Program/PhD Course

Theory of Computation II: COM 5108

Tutorial VII (22 October 2025)

Instructor: Goutam Biswas *Autumn Semester 2025*

Exercise 1. Can the language language $\{1^p : \ p \text{ is a prime}\}$ be decided in $O(\log n)$ space?

Exercise 2. Let w_k is the concatenation of k-bit binary numerals in increasing order and seperated by $*$'s e.g. $w_3 = 000 * 001 * 010 * 011 * 100 * 101 * 110 * 111$. The set $A = \{w_k \in \{0, 1, *\}^* : \ k \in \mathbb{N}\}$.

(a) Show that A is not regular.

(b) Show that A can be decided in $O(\log \log n)$ space.

Exercise 3. (*Satisfiability of Quadratic System over* $\mathbb{F}_2$) Simultaneous solutions of quadratic equations of variables $\{x_1, \cdots, x_n\}$ over $GF(2)(\mathbb{F}_2)$ is **NP**-*complete.*

Definition 1. $SUBSET - SUM = \{< S, t >: \ S = \{x_1, \cdots, x_k : x_i \in \mathbb{N}\}$ is a multiset and for some $\{x_{i_1}, \cdots, x_{i_l}\} \subseteq S$, $\sum_{j=1}^{l} x_{i_j} = t\}$.

Exercise 4. $SUBSET - SUM$ is **NP**-*complete.*